The Primary-Response Framework for Geometallurgical Variables

S Coward[1], J Vann[2], S Dunham[3] and M Stewart[4]

ABSTRACT

Geometallurgy is an emerging, cross-disciplinary field that integrates spatial models of rock properties with time-based outcomes of mining and treatment processes. Geometallurgical variables include any rock property that has a positive or negative effect on the business. Some of the more critical geometallurgical variables include recovery, grindability, throughput, power consumption, mineralogy and content of deleterious materials. These variables drive project costs and revenues in a fundamental way and thus geometallurgy has potential to positively impact on the value of strategic and tactical decisions across many mining organisations.

Geometallurgical models, like traditional grade-only models are based on samples collected from the mineralised and non-mineralised regions of the deposit. The nature of some non-grade variables, particularly those that are dependent on the ore treatment processes, requires special consideration during sampling and subsequent spatial modelling. Failing to objectively consider the characteristics of variables may seriously compromise the validity of the sample selection, sample treatment, data models and the subsequent decisions. In addition to addressing the vexed question of collecting an 'unbiased and representative' sample, the sampling and modelling approach must consider issues of scale, both geostatistical support and also the relationship between bench-scale data and operational-scale mineral processing performance.

This paper proposes a framework (the Primary-Response framework) for the classification of geometallurgical variables. This framework is designed to assist with developing sampling approaches and identifying the most appropriate spatial modelling approach. The proposed framework can also help identify the risks associated with designing, sampling and modelling of both types of geometallurgical variables.

The Primary-Response framework divides variables into two categories; primary variables and response variables. How a variable is classified in this framework depends on the degree to which the variable reflects either an intrinsic attribute of the rock ('primary') or its response to measurement processes ('response'). Depending on the classification in this framework, appropriate sampling and modelling decisions can be made to minimise the risk associated with the incorrect treatment of the variable. In particular, the authors argue that for response variables, the approach taken to building spatial models may need to be different to conventional linear averaging approaches (for example kriging).

INTRODUCTION

The value proposition of geometallurgy is simple and compelling. By improving the understanding of the spatial nature of relevant rock properties the mining and ore treatment operations can be improved, both at the design phase and operation of mineral projects. In theory an exhaustive description of the rock and its performance under the conditions imposed by mining and ore treatment would simplify the problem of predicting physical and financial outcomes by reducing the number of unknowns in the value-generation process.

1. Principal Consultant, Quantitative Group, PO Box 1304, Fremantle WA 6959. Email: sc@qgroup.net.au

2. FAusIMM, Director, Quantitative Group, PO Box 1304, Fremantle WA 6959. Email: jv@qgroup.net.au

3. MAusIMM, Director, Quantitative Group, PO Box 5615, Brendale Qld 4500. Email: sd@qgroup.net.au

4. Principal Consultant, Quantitative Group, PO Box 1304, Fremantle WA 6959. Email: ms@qgroup.net.au

Because analytical and mathematical technology has advanced, metallurgical, mining engineering and geological professionals are now able to measure and manage a much wider range of rock attributes, bringing our knowledge closer to the ideal of 'exhaustive description of the rock'. It follows that creating spatial (block) models of these additional attributes will allow business planners to take advantage of the additional detail to make better decisions about value.

Once generated, spatial geometallurgical models may be used in numerous ways, including improved:

- mine and process design and thus more efficient capital allocation,
- mining project valuation – potentially forming the basis of strategic advantages for early leaders in this field by enabling revaluing of assets,
- processes of predicting and increasing return from more efficient sequencing/scheduling;
- forecasting of revenues and costs,
- process optimisation (with additional benefits from being more proactive), and
- tactical improvements to planning (block selection) and blending strategies in the short to medium term.

The integration of these geometallurgical models into the evaluation and optimisation of reserves is vital to ensure value realisation from geometallurgical initiatives. Useful approaches of including multiple estimates in block models for mine planning and reserve evaluation and have been covered by several authors (Nicholas *et al*, 2006; Nicholas *et al*, 2007; Carrasco, Chilès and Segurét, 2008; Dowd, 1976; Deraisme and Fouquet, 1983) and will not be dealt with in any detail in this paper.

The design and successful execution of an appropriate sampling strategy is the foundation of the modelling and estimation process both for grade models and more sophisticated, multivariate geometallurgical models. Sufficient numbers of samples of appropriate size, ie geostatistical support, are required from major domains in the orebody so that both the average and variability of the geometallurgical variables can be spatially modelled with a known degree of uncertainty. In addition to collecting enough samples, however, the geometallurgical sampling program must also consider the relationship between bench-scale testing and operational-scale performance, and the implications for the minimum mass of samples required for metallurgical tests.

Once the sampling strategy has been implemented, the spatial model can be constructed. As described in Dunham and Vann (2007), applying classical linear averaging resource estimation techniques (including kriging) may not be appropriate. Many of the most important geometallurgical variables are clearly non-additive, therefore the modelling (and subsequent scheduling and analysis) must be appropriately tailored for the nature of the variable in question.

Designing the sampling and spatial modelling approach for some geometallurgical variables can be extremely complex, whereas other variables are much simpler to manage. When possible, distilling the simple variables from the complex variables is beneficial and assists with developing an understanding of the value generation process that is being modelled. One possible pathway for implementing geometallurgical modelling is to estimate, wherever possible, additive variables that enable the prediction of required non-additive properties. To do this effectively, however, requires a framework to classify the geometallurgical variables, a proposed classification system is now discussed.

PROPOSED PRIMARY VERSUS RESPONSE FRAMEWORK

The concept of directly measured variables or proxies for metallurgical performance was discussed in Dunham and Vann (2007). Building on these concepts, the authors now propose a two-fold classification scheme (depicted in Figure 1):

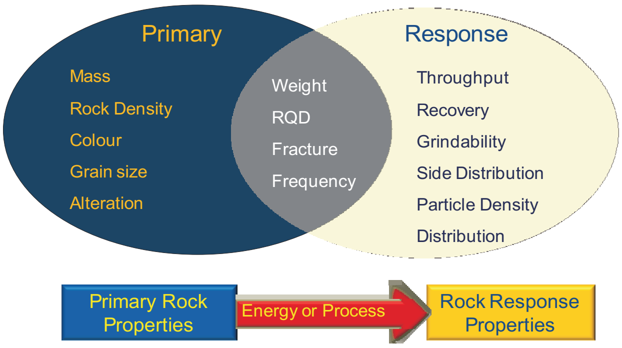

FIG 1 - The Primary-Response framework.

1. primary variables – these are attributes of rock that can be directly measured (metal and mineral grades being examples); and
2. response variables – these are attributes of rock that describe the rock's responses to processes, for example throughput, or recovery.

In our proposed classification scheme, one can consider a property as primary if the property is *intrinsic* to the rock, for example: grain size, mass, metal grades and mineral grades. Most primary variables are additive or can be easily manipulated to be treated as additive, eg metal grades per unit mass, mineral grades per unit volume, etc. The mean of such attributes can usually be correctly estimated by a simple linear average, from both a sampling (compositing) viewpoint as well as in the block-modelling process.

On the other hand, response variables are expressed as a response to a process or the application of energy, for example: grindability, metallurgical recovery, intact rock strength and plasticity. Due to the multivariate nature of these variables the resulting distributions of the measured data can be complex (non-normal, negatively skewed, bi-modal) and hence they cannot easily be combined nor will arithmetic averages produce a valid estimate of the outcome from the combination of a number of samples or blocks. For example, combining one tonne of rock with a measured recovery of 60 per cent with one tonne of rock with a measured recovery of 90 per cent does not necessarily result in two tonnes of rock with a recovery of 75 per cent.

The distinction between primary and response is not always obvious. Take for example the mass of a sample versus the weight of that sample. The mass is a primary characteristic whereas weight is a response to gravitational energy. A sample weight is dependent on the gravitational field (energy). Consider weighing the same sample on the moon!

In an ideal testing framework the response variable would be independent of the testing procedure. This can be considered true, to an extent, for grade assays that theoretically produce results whose mean and variance is essentially a function of the metal contained in the sample treated and not the parameters, or methods used for the assay. In reality, and particularly in the testing of the physical characteristics of rock samples, the result obtained is clearly dependent on the parameters of the testing process.

A primary objective of geometallurgy is to spatially estimate variables into block models, thus the primary-response distinction has important ramifications. The question of support and scale is always critical when consolidating or combining variables. Some variables cannot be linearly averaged. The property that allows the mean of some variables to be calculated by a simple linear average is known as 'additivity'. To legitimately average values of an attribute without generating biases, we must ensure that the attribute we are dealing with is additive. This is true for simple arithmetic averaging, and for other linear combinations, such as weighted averages. Kriging (Matheron, 1963) and other common spatial estimators based on linear averaging all presume additivity of the attribute being estimated. Using estimators that assume additivity for non-additive variables will generate results that are potentially biased in ways that are complex and possibly material. This is also true of non-linear geostatistical estimators such as uniform conditioning, multiple indicator kriging, and other procedures such as conditional simulation which are built upon kriging (Vann and Giubal, 2000).

Response variables are usually complex and are typically non-linear, either through some categorical relationship or through a non-linear formula (Figure 2). This does not suggest that all primary variables are additive, but that they lend themselves to easier assessment of their underlying properties.

The implied non-additivity of response variables can be simply illustrated through Jensen's inequality (Hastings *et al*, 2005). As illustrated in Figure 2, when the relationship between two variables is non-linear, a simple linear average will over- or under-state the true value. The sign of the error (over/under) is dependent on the local behaviour of the non-linear relationship (convex or concave). If the relationship is very complex the prediction errors can also be very complex and difficult to determine.

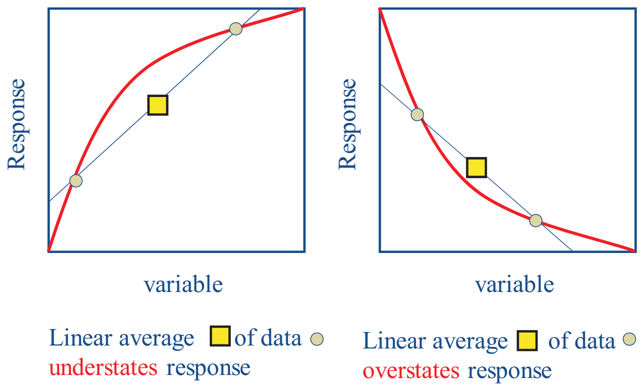

FIG 2 - Jensen's inequality.

CONSEQUENCES OF THE PRIMARY-RESPONSE FRAMEWORK

The Primary-Response classification framework provides an insight into how we should sample and estimate different types of variables. Clearly if we are interested in a primary variable the sampling, estimation and evaluation approach will be much easier than if we are dealing with a response variable where our approach must also incorporate an understanding of both the measurement systems (ie the experimental framework such as parameters in a flotation test) *and* the complexities of the non-additive relationship of the variable. We cannot simply add two float tests together, divide by two and get an 'average flotation' result.

The Primary-Response framework encourages us to identify, measure and model primary variables wherever possible. If we can reduce a response variable down to primary components that (substantially) control the rock behaviour under a given experimental approach we can simplify the problems of sampling, estimation and evaluation (Carrasco, Chilès and Segurét, 2008). If this strategy is adopted then, for example, instead of dealing with two potentially non-additive and complex unknowns (ie the rock *response* and the test/measurement system) we can sample and model the primary variable and then manage the experimental design under a series of different scenarios (eg using different reagents or different grind sizes). This is particularly beneficial when we are later faced with up-scaling the rock response variables and the variability of the parameters of the test used to generate the response. An example follows below.

Water consumption in a treatment process is often not a cost driver, but rather a strategic consideration when water supply is scarce. Ultimate water consumption will be a function of several variables, including rock clay content, the degree to which the clays are altered and the liberation of the clays which in turn is a function of the fineness of the grind. Primary variables such as clay mineral content and alteration state can be estimated into the blocks and then the transfer function that uses the estimate of clay content and resulting grind can be applied to these estimates at the block scale to generate the response variable 'water consumption'.

The variance of gold grade based on small, core-sized samples of a gold deposit will be much higher than the variance of the gold grades obtained from a set of bulk samples from that same deposit. Measurements made on a population of small samples taken from a deposit will exhibit a different variance to the measurements made on a population of large samples taken from the same deposit. It has been shown that the variance of the population is dependent on the scale or support for that population. This is known as the 'support effect' as described by Krige's relationship (Krige, 1951; Matheron, 1963). The same principle is true for both rock responses (ie response variables) *and* the test methodology used to determine those responses (eg a bench-scale flotation test versus a full-scale flotation cell). In the case of the testing methodology the impact of the 'time support' that is used to define the population of the tests also needs to be considered.

The relationship that exists between the primary variables, (that influence the rock response to a given experimental process), and the measured response is materially dependent on the parameters and scale (support; physical and temporal) of the testing process. Hence another aspect to the Primary-Response framework is required. This can be considered as the 'third vertex' of the Primary-Response framework, and is the nature of the process that is used to generate the rock response. Given that the observed variance of the variables is scale-dependent, characterising the covariances between each of these three aspects of the framework (Figure 3)[5] at each scale for which the data is to be used in the planning and estimation process is important. It must be understood that as we change the scale of the rock that we are measuring, not only will the measured

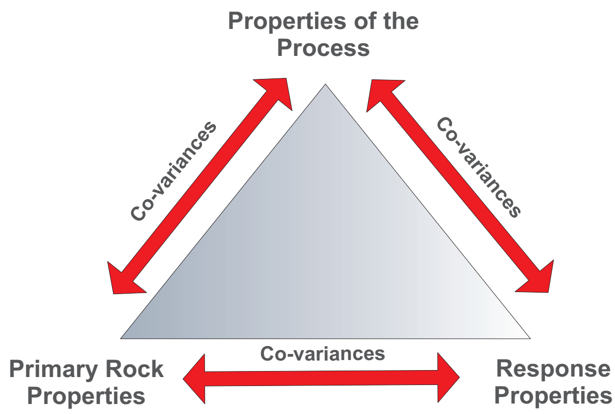

FIG 3 - Covariances in the Primary-Response framework.

attribute change, but the relationship between each of the vertices of the triangle depicted in Figure 3 will also change.

In some situations strong correlations between additive variables may require that their formulation be adapted to ensure they are truly additive. For example metal 'quantity' (per unit volume) and density can usually be dealt with as additive variables, but this assumes that there is little or no correlation between density and grade, eg in low sulfidation gold deposits. However, if there is a strong correlation between grade and density and if density variations are large, as in many base metal deposits, special attention needs to be paid to the way in which the grade variable is expressed and how it is averaged. In this specific case the problem can be overcome by converting the volumetric grades should be converted to total weight of gold in each block and then expressing the grades as mass of gold per tonne of rock.

The Primary-Response framework provides a useful way of classifying rock variables, but its true value lies in providing a framework for coherent and structured investigation of the underlying nature of the variables that are being measured and modelled. This understanding will hopefully reduce the probability of making errors, inform sampling strategy, and increase the value derived from geometallurgy.

SELECTION OF GEOMETALLURGICAL VARIABLES

In an ideal world the geometallurgical model would contain accurate, precise estimates at a block scale of those rock properties impacting most on the value of a project. These properties would be additive and linear in nature and thus be straightforward to estimate by kriging and other linear averaging methods (Dunham and Vann, 2007). Ideally, the rock properties would be primary under the classification proposed above and thus be independent of changes either to the mining method or to the process plant configuration. Mining method and process plant configuration could be directly incorporated in the mining block model and translated into process response independently of the changes in the resource block model, cut-off policy, and/or the mine or plant design. This ideal is not feasible in practice but provides a good target. Deviations from this ideal, by attempting to directly estimate response variables for example, necessitates an acute awareness of the implicit assumptions made and the risk and implications that they have on the use, and hence value, of the model. In reality, this may be difficult to demonstrate in a resource model that is to be independently audited.

All scientific sampling should have the end use of the data in mind. In the case of geometallurgical data, the variables that impact most on the processes of mining and treatment (either positively or negatively) must be identified to determine the

5. Generally the concept of correlation is well understood and covariance is merely a scaled version of correlation. Pearson's correlation coefficient between two variables x and y being the covariance divided by the product of the standard deviation of x and the standard deviation of y. Thus Pearson's correlation coefficient is a rescaled covariance to the interval [-1,1].

variables that need to be measured. The required variables will be specific not only to commodity and deposit type, but also potentially to the mining and processing technologies envisaged. There are a number of ways to identify and then target and prioritise the required variables in the sampling strategy. A number of possible approaches to geometallurgical variable selection are now considered below.

In a 'brownfields' case, identifying and modelling of geometallurgical variables is greatly assisted by the fact that an existing processing plant can provide us with data that can be used to assess the effectiveness of both the geometallurgical sampling and the models we build upon that sampling. Geometallurgical variable selection can be driven by analysis of existing mine performance, in practice by relating periods of specific metallurgical performance to specific rock properties. This requires good management of both plant performance data and mine depletion records and therefore is greatly aided by a robust reconciliation systems. One implication of the increased use of geometallurgy is that operational reconciliation systems will become more critical, and must be focused on a broader array of variables than the traditional tonnes and grade.

Because many mines treat more than one ore source at a time, reconciling process performance back to ore feed properties and then tracking that back to *in situ* locations and lithology types is never straightforward. Such reconciliation, even if incomplete, can be of great use in identifying areas of importance and the potential predictive power of different geometallurgical variables. Where the model and reality diverge, work can be undertaken to understand the reasons for the discrepancy. In fact, such reconciliation discrepancies have been used to factorise the models to assist in short-term forecasting.

The authors point out that such discrepancy may arise out of the fact that linear models are being assumed for variables with non-linear behaviours. There is therefore some attendant risk in factorising models to account for non-linearity. To illustrate this point Figure 4 shows a diagrammatic representation of Jensen's inequality, where the solid curve and dashed dotted line represent the true (solid) and modelled (dashed dotted) relationships between a grindability index and the percentage recovery. The model is based on an assumed linear relationship derived from two sample points (indicated by two squares) whereas the underlying true relationship is indicated by a curve that is concave to the origin (the solid curve). In order to get the linear model to 'work' or to fit data obtained from the current operation a so called 'efficiency factor' of just over ten per cent has been added (ie factor the linear model by about ten per cent).

If, for example due to production pressure, it is decided to move to an area of the mine that has a better grindability index, it would be expected, based on our linear model and the additional impact of the 'efficiency factor', that recovery would increase to the high 90 per cent's. In reality a figure in the high 80 per cent's is more likely due to the *non-linear* relationship. Had sufficient sampling been carried out to model the true underlying primary-response behaviour the mistaken high expectation may have been avoided.

The point made in the above example is that when sampling for geometallurgical response variables we not only have to characterise the mean and variance of the variable but also the nature of the *non-linear* relationship of the primary and response variables. Assuming linear relationships between primary and response variables is risky and can lead to significant error. This is the case for simple bivariate relationships and will also be the case for spatial interpolation. Furthermore, simple (and simplistic) factoring of the results may apparently correct the results over a limited range of values, but could cause unintended biases away from that range.

For 'greenfields' projects selecting the right geometallurgical variables is a little more difficult. In this case the processing options must be clarified, and then inferences about ore/process impacts might be based on assessment of a matrix of deposit/process combinations that are similar. This approach may necessitate collection of more than one suite of samples to characterise geometallurgical variables; for example, in a copper deposit, some samples will be required and be prepared specifically for conventional flotation circuits and others required for assessing leaching options.

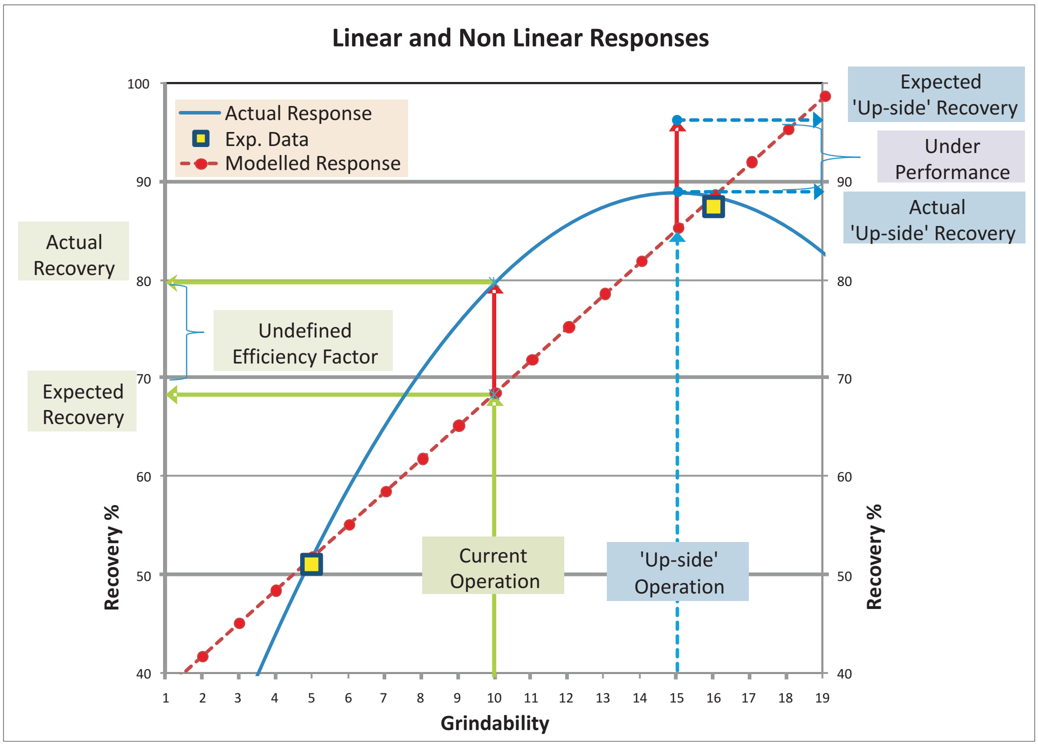

FIG 4 - Assuming an 'undefined efficiency factor' to account for the non-linear relationship between grindability and recovery, can lead to a false expectation of upside recovery at higher grindability.

Even in the case of a single processing option we must take care to consider that there will usually be various processing options for the same technology (grind-recovery relationships for conventional flotation processing being an example). It is also worth exploring the role that metallurgical process simulation can play in unravelling these complex relationships, and as each of the process models may require calibration through sampling and test work, additional material may be specifically required.

SAMPLE MEASUREMENT AND METROLOGY

Once samples have been obtained and in some cases divided into different portions (ie for different types of analysis and test work), they will be dispatched to various laboratories. As usual due care needs to be taken to ensure that they are not mishandled or allowed to become contaminated either with other samples or through contact with other reagents that might compromise the results that are obtained. For geometallurgy, especially physical testing it is important that the physical integrity be preserved. As an example kimberlite typically contains several species of swelling clays. Hence the samples need to be wiped dry immediately after drilling, and packed and kept dry until tested, otherwise measurements of rock strength and degree of alteration will not reflect what will be encountered when mining.

For geologists who may be used to only requesting grade assaying, it is worth noting that the numerous geometallurgical tests that can be performed have rigorous requirements for calibration. This is especially true of mass measurement, and screening procedures which should be repeatedly calibrated, and ideally certified, with results of calibration checks being recorded.

It has been shown (Stark, Perkins and Napier-Munn, 2008; Napier-Munn *et al*, 1999) that the potential for operator error in physical testing is substantial. This suggests that any sampling procedure should minimise factors that depend on operator input. A good example of this is flotation test work, where historically, operator influences can be significant. Consistency of test work is very important. The relationship between primary and response variables is strongly influenced by the testing regime therefore if the testing approach is inconsistent our ability to model response variables from primary variables is greatly reduced.

CONCLUDING REMARKS

A spatial model containing valid estimates of the geo-metallurgical variables provides a platform for improving mining project return in many different ways. Acquisition of the right number and size of samples that can be appropriately tested to provide the right data for the spatial estimation of these variables is perhaps one of the greatest challenges faced in the generation of the spatial geometallurgical model.

The Primary-Response framework as presented here provides a useful basis upon which a strategy for sampling design and execution can be built. It is advisable to think through the geometallurgical modelling requirements prior to design of sampling, and the Primary-Response framework can help in this regard.

Since primary variables are in general additive, routine linear averaging estimators such as kriging seem appropriate. However, direct estimation of response variables by linear averaging has potential pitfalls. A preferred approach is to estimate primary variables wherever possible. Note that if the range of values for which response variables are required is relatively small, the non-linearity may not have a material effect.

In addition to the usual good practices in sampling, geometallurgy also requires that the physical integrity of samples be carefully managed in many instances. The range of processes, ore responses, and testing methods can be large and the options considered may have differing geometallurgical sampling and modelling requirements.

Insufficient or incorrect information on the spatial location and variability of the rock properties that impact most on the process will expose mining operations to unnecessary risk. The effort and time required to acquire the data to build valid geometallurgical models that can be integrated into the decision-making process throughout the mine life cycle will generate rewards that are many times larger than the costs incurred.

REFERENCES

Carrasco, P, Chilès, J P and Segurét, S, 2008. Additivity, metallurgical recovery and grade, in *Proceedings Eighth International Geostatistical Congress*, Santiago, 1188:237-246.

Deraisme, J and Fouquet, C, 1983. Recent and future developments in ‘downstream’ geostatistics, in *Proceedings NATO ASI Geostatistics for Natural Resources Characterisation* (eds: G Verly, M David, A G Journel and A Marechal), pp 979-999 (Reidel Publishing Company: Dordrecht).

Dowd, P A, 1976. Application of dynamic and stochastic programming to optimise cut off grades and production rates, *Transactions of the Institutions of Mining and Metallurgy*, Mining Technology, 85:A22-A31.

Dunham, S and Vann, J, 2007. Geometallurgy, geostatistics and project value – Does your block model tell you what you need to know?, in *Proceedings Project Evaluation 2007*, pp 189-196 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Hastings, A, Arzberger, P, Bolker, B, Collins, S, Ives, A, Johnson, N and Plamer, M, 2005. Quantitative bioscience for the 21st century, *Bioscience*, 55(6):511-517.

Krige, D G, 1951. A statistical approach to some basic mine valuation problems on the Witwatersrand, *Proceedings Chemical, Metallurgical and Mining Society of South Africa,* 52(6):119-139.

Matheron, G, 1963. Principles of geostatistics, *Economic Geology,* 58(8)1246-1266.

Napier-Munn, T, Morell, S, Morison, R and Kojovic, T, 1999. *Mineral Comminution Circuits, Their Operation and Optimisation*, second edition, 413 p (JKMRC, University of Queensland: Indooroopilly).

Nicholas, G D, Coward, S J, Armstrong, M and Galli, A, 2006. Integrated mine evaluation – Implications for mine management, in *Proceedings International Mine Management Conference,* pp 69-79 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Nicholas, G D, Coward, S J, Rendall, M and Thurston, M L, 2007. Decision-making using an integrated evaluation model versus sensitivity analysis and Monte Carlo simulation, in *Proceedings CIM International* (Canadian Institute of Mining Metallurgy and Petroleum: Montreal).

Stark, S, Perkins, T and Napier-Munn, T, 2008. JK drop weight parameters – A statistical analysis of their accuracy and precision and the effect on SAG mill comminution circuit simulation, in *Proceedings MetPlant 2008*, pp 147-156 (The Australasian Institute of Mining and Metallurgy: Melbourne).

Vann, J and Giubal, D, 2000. Beyond ordinary kriging: An overview of non-linear estimation, in *Mineral Resource and Ore Reserve Estimation: The AusIMM Guide to Good Practice,* pp 249-256 (The Australasian Institute of Mining and Metallurgy: Melbourne).